CALCULATION OF AXIALLY SYMMETRICAL FREE EXPANSION OF A NONEQUILIBRIUM HYDROGEN PLASMA

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A scheme and the results of a calculation by the method of characteristics are presented for free expansion of a nonviscous, thermally nonconducting, nonequilibrium, optically thick hydrogen plasma from a round supersonic nozzle. The elementary process determined is considered to be collision-radiative recombination. A strong disturbance in the thermal and ionization equilibrium are observed in the flow field. The effect of relaxation processes on the geometry of flow and the field of gas-dynamic parameters is examined. The results of the calculations are compared with analogous data for an ideal perfect gas.

1. Basic Assumptions

The one-dimensional free expansion of a nonequilibrium plasma in a spherical source was examined in [1-3]. In the present work the axially symmetrical stationary problem of the free expansion of a nonviscous, thermally nonconducting, relaxing, optically thick hydrogen plasma is solved on the basis of the macroscopic approach. The choice of hydrogen as the subject of the study was dictated by considerations of simplicity and clearness. In the calculation the following assumptions are assumed to be valid:

1) the plasma consists of electrons, ions, and atoms;

2) the elementary process which is determined is collision-radiative recombination; acts of ionization are absent at the mouth of the nozzle;

3) the velocity of all the plasma components is the same;

4) the plasma is quasineutral;

5) the internal energy of the electron component is determined by a balance between the energy lost by electrons in elastic collisions with heavy particles and the energy received by electrons during recombination [2];

6) the coefficient of recombination is a single-valued function of the local temperatures and electron concentrations [4].

2. System of Equations

With the assumptions made the system of equations describing the axially symmetrical free expansion of the plasma has the form

$$\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} + \rho \frac{v}{y} + \frac{d\rho}{dt} = 0 \tag{2.1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \tag{2.2}$$

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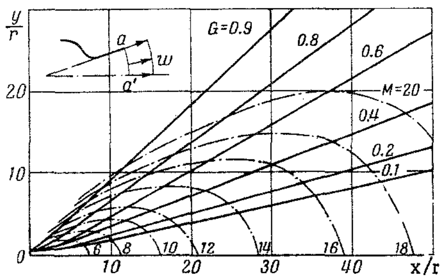

Fig. 1

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial y}=0 \tag{2.3}$$

$$\frac{dh}{dt}=\frac{1}{\rho}\frac{dp}{dt} \tag{2.4}$$

$$h(T,T_e,\alpha)=\frac{1}{m_a}\left[\frac{5}{2}k(T+\alpha T_e)+\alpha I\right] \tag{2.5}$$

$$\frac{d\alpha}{dt}=F_\alpha=-\frac{\rho}{m_a}K_r(N_e,T_e)\alpha^2 \tag{2.6}$$

$$Q^Y+Q^P=0 \tag{2.7}$$

$$Q^Y=\frac{N_e^2e^4}{m_a}\left(\frac{8\pi m_e}{kT_e}\right)^{1/2}\left(\frac{T}{T_e}-1\right)\ln\left[\frac{9(kT_e)^3}{8\pi N_e e^6}\right]$$

$$Q^P=\frac{I+{}^5/_2kT_e}{m_a^2}K_r(N_e,T_e)\alpha^2\rho^2$$

$$p=\frac{k}{m_a}\rho(T+\alpha T_e) \tag{2.8}$$

$$\alpha=\frac{m_a}{\rho}N_e,\quad \rho=m_a(N_e+N_a)$$

Here x and y are the coordinates (the x axis is directed along the axis of symmetry); u and v are projections of the velocity on the x and y axes, respectively; ρ is the density; p is the pressure; h is the enthalpy; T and T_e are the temperatures of the heavy particles and electrons; α is the degree of ionization; N_a and N_e are the concentrations of atoms and electrons; m_a and m_e are the atomic and electron masses; e is the electron charge; Q^Y is the energy lost by electrons during elastic collisions with ions (collisions with atoms at appreciable degrees of ionization are not significant because of the small collision cross sections); Q^P is the energy transferred to electrons during recombination [1]; I is the ionization potential; K_r is the collision-radiative recombination coefficient; k is the Boltzmann constant.

The energy equation for the electron component (2.7) is an approximative equation. It is valid in some region of flow with a sufficiently high density of charged particles, when the work of expansion is negligibly small in comparison with Q^Y and Q^P [2]. In carrying out specific calculations the validity of the equations of electron energy written in the form (2.7) should be confirmed by direct quantitative estimates.

3. Equations of Characteristics

The method of characteristics in the form proposed in [5] can be used for a numerical solution of the system of equations (2.1)-(2.8). The calculating system of equations includes equations of characteristics of the first and second groups and correlations along the current lines. The equations of characteristics are written in the form

$$dx=\frac{\beta\mp\zeta}{\beta\zeta\pm1}dy \tag{3.1}$$

$$\frac{1}{1+\zeta^2}d\zeta\pm\frac{\beta}{\rho w^2}dp\pm\frac{1}{\beta\zeta\pm1}\left\{\frac{\zeta}{y}+\frac{(1+\zeta^2)^{1/2}IF_\alpha}{{}^5/_2kw(T+\alpha T_e)}\right\}dy=0 \tag{3.2}$$

$$d\psi=\pm\frac{\rho wy(1+\zeta^2)^{1/2}}{\beta\zeta\pm1}dy \tag{3.3}$$

Here $\zeta=\tan\theta$ (θ is the angle between the velocity vector and the x axis); $\beta=\sqrt{w^2/a^2-1}$, $w=\sqrt{u^2+v^2}$, and ψ is a function of the current; a is the "frozen in" speed of sound and $a^2=5k(T+\alpha T_e)/3m_a$.

The correlations along the current lines have the form

$$dy=\zeta dx \tag{3.4}$$

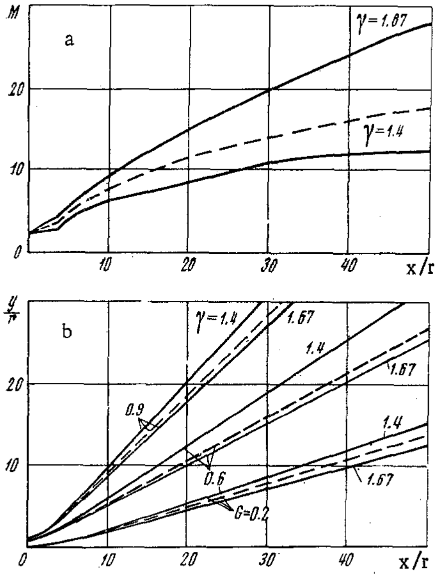

Fig. 2

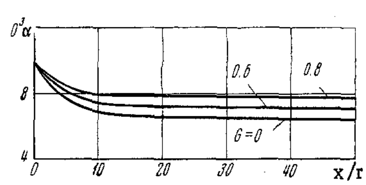

Fig. 3

$$d\alpha = \frac{(1+\zeta^2)^{1/2}}{w} F_a\, dx \tag{3.5}$$

$$\frac{3}{2}\frac{dp}{\rho} - \frac{5}{2}\frac{p}{\rho^2}\, d\rho + \frac{I}{m_a}\, d\alpha = 0 \tag{3.6}$$

$$\frac{w^2}{2} + h = h_0, \quad h = \frac{1}{m_a}\left[\frac{5}{2}k(T + \alpha T_e) + I\alpha\right] \tag{3.7}$$

$$Q^Y + Q^P = 0 \tag{3.8}$$

$$p = \frac{k}{m_a}\rho(T + \alpha T_e) \tag{3.9}$$

4. Schema and Example of Calculation

For the calculations the system of equations (3.1)-(3.9) is written in the finite-difference form in accordance with [5]. The program of flow calculation includes a subprogram for calculating the parameters at the initial surface, in the field of flow, on the axis of the jet, and in the vicinity of the angle point (nozzle rim). The parameters of flow at the nodes of the network of characteristics are determined by the method of successive approximations. The most slowly falling value is the temperature of the electrons. This value was also chosen as the criterion of convergence. The calculation of the parameters at each point was stopped when the difference in values of T_e for two successive approximations was $10^{-3}\%$. An average of five to six approximations was made in calculating each point.

As an example let us examine the free expansion from a conical nozzle with a semiaperture angle of 11.3° ($\zeta = 0.2$). We assume that the geometry of flow at the mouth of the nozzle corresponds to flow at a source with a pole at the point of intersection formed by the nozzle with the axis of symmetry (Fig. 1). The parameters are assumed to be constant at the surface aa'. The calculation was carried out for a radius r of the exit section of the nozzle equal to 1 cm, and for the following parameters at the mouth of the nozzle:

$$M = w/a = 2.5,\ w = 1.4\cdot 10^6\,\text{cm/sec},\ T = 2\cdot 10^{3\circ}\,\text{K}$$

$$\rho = 5\cdot 10^{-8}\ \text{g/cm}^3$$
$$p = 8.3\cdot 10^{-3}\ \text{kg/cm}^2$$

The values of α and T_e at the initial surface are found from Eqs. (3.7) and (3.9). The collision-radiative recombination coefficients are taken from [4].

The results of a calculation of the numbers M and the spectrum of current lines are presented in Fig. 1. On the current lines are given values of the relative mass flow rates G which pass through within a body of rotation formed by the given current line. The axis of symmetry corresponds to G = 0 and the boundary of flow to G = 1. The results of a calculation of the M numbers along the jet axis of a hydrogen plasma (dashed line) are compared with the M numbers along the axis of jets of ideal perfect monoatomic ($\gamma = 1.67$) and diatomic ($\gamma = 1.4$) gases in Fig. 2a. The energy release in a recombining expanding plasma leads to a decrease in the M numbers in the field of flow and a stronger turning of the gas away from the axis of symmetry (Fig. 2b). In the example under consideration the effect of relaxation processes on the geometry of flow and the field of w and ρ is not great, which is explained by the delayed transfer of ionization energy into progressive degrees of freedom. For the parameters selected the ionization energy at the mouth of the nozzle makes up ~20% of the total enthalpy. However, because of the rapid "freezing in" of the degree of ionization along the current lines (Fig. 3), only a small part of the stored energy is released. The difference in the trend of α along different current lines is connected with the difference in the rate of expansion of the plasma along these lines.

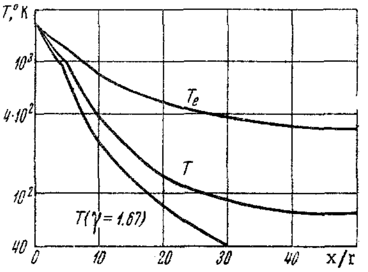

Fig. 4

A disturbance in the thermal equilibrium is typical for a freely expanding recombining plasma because of the delayed exchange of energy between the electrons and heavy particles. The results of a calculation of T and T_e along the axis of symmetry are presented in Fig. 4. The ratio T_e/T increases downstream. Quantitative estimates show that the energy equation for the electron component in the form (2.7) can be used in the entire region of the calculation.

With an increase in the degree of ionization at the mouth of the nozzle and a sufficiently high rate of recombination the effect of relaxation processes on the geometry of flow becomes even more noticeable. It should be noted, however, that for significant values of α the approximative approach used becomes inadequate. The increase in the role of radiation and relaxation processes requires a more detailed statement of the problem, taking into account the kinetics of the population of discrete energy states and the diffusion of radiation.

LITERATURE CITED

1. N. M. Kuznetsov and Yu. P. Raizer, "Recombination of electrons in a plasma expanding into a vacuum," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 4 (1965).
2. G. A. Luk'yanov, "A stationary supersonic source of a nonequilibrium plasma," Ibid., No. 6 (1968).
3. Y. S. Chou and L. Talbot, "Source-flow expansion of a partially ionized gas into a vacuum," AIAA Journal, 5, No. 12 (1967).
4. D. R. Bates, A. E. Kingston, and R. W. P. McWhirter, "Recombination between electrons and atomic ions II," Proc. Roy. Soc., Ser. A, 270, No. 1341 (1962).
5. O. N. Katskova and A. N. Kraiko, Calculation of Planar and Axially Symmetrical Supersonic Currents in the Presence of Irreversible Processes [in Russian], VTs Akad. Nauk SSSR, Moscow (1964).